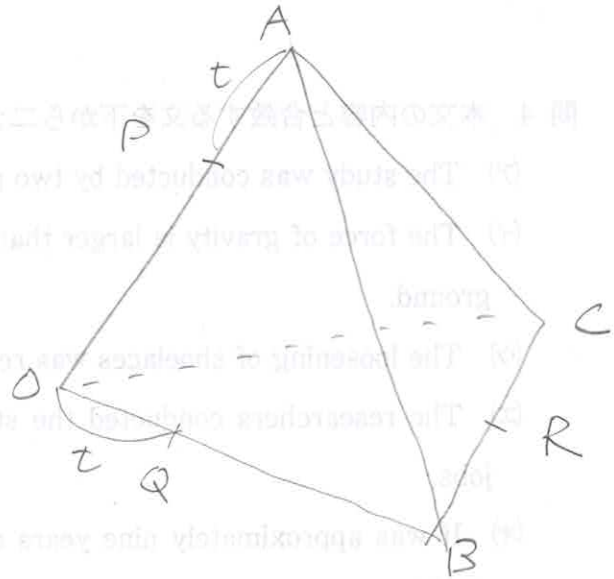


1.



$$(1) \vec{QP} = \vec{OP} - \vec{OQ} = (1-t)\vec{a} - t\vec{b}$$

$$\vec{QR} = \vec{OR} - \vec{OQ} = \frac{\vec{b} + \vec{c}}{2} - t\vec{b} = \left(\frac{1}{2} - t\right)\vec{b} + \frac{1}{2}\vec{c}$$

$$(2) |\vec{a}| = |\vec{b}| = |\vec{c}| \quad \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \frac{1}{2}$$

$$\begin{aligned} \vec{QP} \cdot \vec{QR} &= (1-t)\left(\frac{1}{2} - t\right) \cdot \frac{1}{2} + (1-t) \cdot \frac{1}{2} \cdot \frac{1}{2} - t\left(\frac{1}{2} - t\right) - \frac{t}{2} \cdot \frac{1}{2} \\ &= \frac{1}{4}(1 - 3t + 2t^2 + 1 - t - 2t + 4t^2 - t) \end{aligned}$$

$$\vec{QP} \cdot \vec{QR} = \frac{1}{4}(6t^2 - 7t + 2) = \frac{1}{4}(2t - 1)(3t - 2)$$

$$\therefore t = \frac{1}{2}, \frac{2}{3}$$

(3) $t = \frac{1}{2}$ のとき

$$\vec{QP} = \frac{1}{2}\vec{a} - \frac{1}{2}\vec{c} \quad |\vec{QP}| = \sqrt{\frac{1}{4} - 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4}} = \frac{1}{2}$$

$$\vec{QR} = \frac{1}{2}\vec{c} \quad |\vec{QR}| = \frac{1}{2} \quad \therefore \Delta PQR = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$t = \frac{2}{3}$ のとき

$$\vec{QP} = \frac{1}{3}\vec{a} - \frac{2}{3}\vec{b} \quad |\vec{QP}| = \sqrt{\frac{1}{9} - 2 \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{2} + \frac{4}{9}} = \frac{\sqrt{3}}{3}$$

$$\vec{QR} = -\frac{1}{6}\vec{b} + \frac{1}{2}\vec{c} \quad |\vec{QR}| = \sqrt{\frac{1}{36} - 2 \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4}} = \frac{\sqrt{7}}{6}$$

$$\Delta PQR = \frac{1}{2} \cdot \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{7}}{6} = \frac{\sqrt{21}}{36}$$

2. (1) 点 $(t, f(t)) = (t, (2t-1)^3)$

接線の傾き $f'(t) = 6(2t-1)^2$

接線: $y - (2t-1)^3 = 6(2t-1)^2(x-t)$

$y=0$ のとき $-(2t-1)^3 = 6(2t-1)^2(x-t)$

$-(2t-1) = 6(x-t)$

$x = \frac{-2t+1}{6} + t = \frac{4t+1}{6}$

(2) $x_1 = 2, x_{n+1} = \frac{4x_n+1}{6} \rightarrow 6x_{n+1} = 4x_n+1$

$6x_{n+1}-3 = 4x_n-2 \quad x_{n+1} - \frac{1}{2} = \frac{2}{3}(x_n - \frac{1}{2})$

$x_{n+1} - \frac{1}{2} = (\frac{2}{3})^n \cdot \frac{3}{2} \quad x_n = \frac{3}{2}(\frac{2}{3})^{n-1} + \frac{1}{2}$

$\frac{3}{2}(\frac{2}{3})^{n-1} > 0 \quad \therefore x_n > \frac{1}{2}$

(3) $|x_{n+1} - x_n| = -(\frac{3}{2}(\frac{2}{3})^n + \frac{1}{2}) + (\frac{3}{2}(\frac{2}{3})^{n-1} + \frac{1}{2}) = \frac{1}{2}(\frac{2}{3})^{n-1}$

$\frac{1}{2}(\frac{2}{3})^{n-1} < \frac{3}{2^2} \times 10^{-5}$

$\frac{2^{n-2}}{3^{n-1}} \cdot \frac{2^2}{3} = (\frac{2}{3})^n < 10^{-5}$

$n(\log 2 - \log 3) < -5$

$n-1 > \frac{-5}{\log 3 - \log 2} \approx 28.4$

$(0.175 < \log 3 - \log 2 < 0.177)$

最小の n は 29

3. (1) $x=1$ を代入 $x^2 - a + b = 0 \quad \therefore a = b + 1$

$(a, b) = (3, 2), (4, 3), (5, 4), (6, 5), (7, 6), (8, 7)$

$$\frac{10}{6 \cdot 6 \cdot 6} = \frac{5}{108}$$

(2) 解を a, β とおくと

$$\begin{aligned} a + \beta &= a & a\beta &= b & a, b > 0 & \text{且} \\ & & & & \alpha > 0, \beta > 0 & \end{aligned}$$

$$1 \leq a \leq 6, \quad 2 \leq b \leq 12 \quad \text{且} \quad |b|$$

$a=3$ のとき $b=2, (\alpha, \beta) = (1, 2), (2, 1)$

$a=4$ のとき $b=3, (\alpha, \beta) = (1, 3), (3, 1)$

$b=4, (\alpha, \beta) = (2, 2)$

$a=5$ のとき $b=4, (\alpha, \beta) = (1, 4), (4, 1)$

$b=6, (\alpha, \beta) = (2, 3), (3, 2)$

$a=6$ のとき $b=5, (\alpha, \beta) = (1, 5), (5, 1)$

$b=8, (\alpha, \beta) = (2, 3), (3, 2)$

$b=9, (\alpha, \beta) = (3, 3)$

(3) (2) 且 整数解を求めるとき

$(a, b) = (3, 2), (4, 3), (4, 4), (5, 4), (5, 6), (6, 5), (6, 8), (6, 9)$

$$1 + 2 + 3 + 3 + 5 + 4 + 5 + 4 = 27$$

$$\frac{27}{216} = \frac{1}{8}$$